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LOW FREQUENCY SOUND RADIATION FROM SLENDER BODIES OF REVOLUTION--ETC(U)
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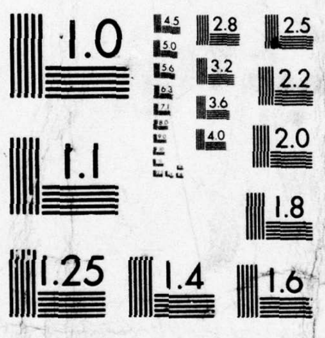

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USL Report No. 706

Low Frequency Sound Radiation from Slender Bodies of Revolution

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7 February 1966

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[illegible] revolution is expressed in terms of a distribution of sources and doublets along the body axis. The strength of the singularities is determined from an analysis of the flow near the slender body. For axiallysymmetric flow a longitudinal rigid body vibration and a simple type of accordion vibration are considered. For these examples the source distribution has the dominant effect on the farfield pressure. For transverse vibration there is only a doublet distribution. The strength of the doublet distribution depends on the force the body exerts on the fluid. For wavelengths much greater than the maximum body diameter, this force can be conveniently determined by the extended Lagally theorem of incompressible hydrodynamics. Formulas for the farfield pressure for each type of vibration are given for spheroids and a simple class of streamline bodies. Examples illustrating the effect of change in body shape and type of vibration are given.

ADMINISTRATIVE INFORMATION

This study was performed under USL Problem No. 1-000-00-00 Bureau of Ships Subproject and Task No. SR 011 01 01-0401.

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NOMENCLATURE

A_s	constant
a	half-length of body
$a_0(x_1)$	horizontal displacement due to vibration of a point on body surface
b	radius of body at $x_1 = 0$
$b(x_1)$	function defining meridian profile
b_1	a constant in equation for streamline body family (Eq. (55))
$\tilde{b}_1$	$= b_1/b$
C_c, C_s	non-dimensional pressure coefficients (Eq. (58))
$\tilde{C}_c$, $\tilde{C}_s$	defined as C_c, C_s for $\cos\theta_3 = 1$
c	the speed of sound for the fluid
D	$= [M(x_1 - \xi_1) + R]/\beta^2$
D_0	$= [M(x_1 + R_0)]/\beta^2$
D_1	$= [M(x_1 - \xi_1) + R_1]/\beta^2$
d	maximum diameter of body
F_n	force normal to surface of body
F_i	$i, = 1, 2, 3$ components of force
f	$= r - b(x_1^*)$ see Eq. (23)
i	$\sqrt{-1}$
J_s^+	defined by Eq. (65)
J_s^-	defined by Eq. (66)
$j_s(x_1)$	spherical Bessel functions of order s
k	$= 2\pi/\lambda$
k_1	$= 1/\beta^2\ k$
ℓ	length of body
ℓ_i	$i = 1, 2, 3$ direction cosines
M	$= U_0/c$
m	source strength
m_0	$= -U_0 S'(x_1)$
m_1	real part of m
m_2	imaginary part of m
n	parameter in Eq. (22)
$\partial/\partial n$	derivative is the direction of the outward normal to surface
O(x)	$y = O(x)$ means $\lim_{x \to o} y/x$ is bounded or zero
$P_s(x)$	Legendre polynomial of order s
$\mathcal{P}_s$	function defined by Eq. (50)
p	farfield pressure
p_0	pressure in undisturbed fluid
Q_s	function defined by Eq. (51)
R	$= \sqrt{(x_1 - \xi_1)^2 + \beta^2 [(x_2 - \xi_2)^2 + (x_3 - \xi_3)^2]}$
R_0	$= \sqrt{x_1^2 + \beta^2 r^2}$
R_1	$= \sqrt{(x_1 - \xi_1)^2 + \beta^2 r^2}$
r^2	$= x_2^2 + x_3^2$
S	surface of body
$S(x_1)$	$= \pi b^2 (x_1)$
s	subscript giving order of Legendre polynomial or Bessel function
t	time
U_0	velocity of uniform streams
u	$= ka \cos\theta_0$
v	$= n\pi/2$
W	transverse velocity of a section of body
W_1	constant in Eq. (60)
W_2	constant in Eq. (60)
w_n	fluid velocity normal to surface
w_r, w_θ, w_1	components of velocity (Eq. (25))
w_1, w_2, w_3	components of velocity

NOMENCLATURE (Cont'd)

x_1, x_2, x_3	rectangular coordinates
x_1^*	$= x_1 - i a_0(x_1) e^{-ikct}$ (Eq. (23))
$\tilde{x}_1$	$= x_1/a$
β^2	$= 1 - M^2$
Γ	average farfield intensity
ϵ_0	error (Eq. (15))
Θ	cylindrical coordinate (Eq (23) and Fig. 1)
θ_0	$= \cos^{-1} x_1/R_0$
θ_2	$= \cos^{-1} x_2/R_0$
θ_3	$= \cos^{-1} x_3/R_0$
λ	wavelength
μ_n	strength of a doublet with axis in a given direction n
$\mu_i(x_1)$	i = 1, 2, 3 strength per unit length of the *ith* component of the doublet distribution (Eq. (41))
ξ_i	i = 1, 2, 3 coordinates of source or doublet
ρ_0	mass density in undisturbed fluid
Φ	velocity potential of the disturbance caused by the body
Φ_T	$= -U_0 x_1 + \Phi$
Φ_s	defined by Eq. (3)
Φ_D	defined by Eq. (4)
Φ_o	defined by Eq. (24)
ϕ	is such that $\Phi = \phi e^{-ikct}$ (Eq. (5))
Ω	$= \cos^{-1} x_3/r$
Ω_1, Ω_2	constants in Eq. (60)
$(\ )'$	$= d(\ )/dx_1$

LOW FREQUENCY SOUND RADIATION FROM SLENDER BODIES OF REVOLUTION

INTRODUCTION

Since the low frequency sound produced by surface ships and submarines can propagate over long distances with small loss of energy, it is of interest to obtain some simple examples of the radiation field of such bodies. In this report the vibrating body is considered to be in an unbounded fluid, and thus the effect of the free surface and bottom (or other external boundary) is not considered. If the body possesses a steady velocity along its longitudinal axis, the radiated field for harmonic vibrations is given for the linearized problem by a formula which is similar to the Helmholtz formula and reduces to that formula when the steady velocity is zero. The results are restricted to the farfield that is due to the vibration of a body of revolution. The body length and the wavelength of the vibration are much greater than the maximum body diameter, and the steady speed of advance of the body is much less than the speed of sound in the fluid. Under these conditions the formula for the radiation field reduces to an integral over a distribution of acoustic sources and doublets along the axis of the body.

For the axially symmetric longitudinal vibrations of a body of revolution only the source term is important. For transverse vibrations the farfield is determined by a doublet distribution. It is shown that, for the low frequency transverse vibration of a slender body of revolution, the force which determines the doublet strength is conveniently given by the extended Lagally theorem of incompressible hydrodynamics. This theorem determines the forces, moments, and added masses for a body which can be represented by a closed stream surface surrounding a distribution of singularities in a potential flow. The theorem was given in 1922 by Lagally for steady flow.[1] Then, in 1953, it was extended to non-steady flow by Cummins.[2]

Related discussions of the low frequency sound radiation from slender bodies have been presented by Strasberg[3] and Chertock.[4,5] However, their examples are restricted to the spheroid. For the spheroid the results given by Strasberg agree with those of the present report. Strasberg, in turn, showed that his results were in agreement with those of Chertock for slender spheroids.

If a given streamline body of revolution does not differ too much from a spheroid, the known results for a spheroid could be used as a first estimate of the radiated field of the streamline body. However, the fact that the streamline body is not symmetric about its midship section introduces effects which are not predicted from a spheroid. To illustrate this, examples are given of the radiated fields of a simple class of streamline bodies and of spheroids with the same length and diameter.

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[1]M. Lagally, "Berechnung der Kräfte und Momente die strömende Flüssigkeiten auf ihre Begrenzung ausüben," Z. angew. Math. Mech. 2, (1922) pp. 409-422.

[2]W. E. Cummins, "The Forces and Moment on a Body in a Time-Varying Potential Flow," Journal of Ship Research 1, No. 1, 7-18 (1957). (First published as David Taylor Model Basin Report No. 780, June 1953.)

[3]M. Strasberg, "Sound Radiation from Slender Bodies in Axisymmetric Vibration," Paper O-28 in Proceedings of the 4th International Congress on Acoustics, 1962, Copenhagen (Organization Committee of the 4th ICA and Harlang & Toksvig, Copenhagen, 1962).

[4]G. Chertock, "Effects of Underwater Explosions on Elastic Structures," Fourth Symposium on Naval Hydrodynamics, B. L. Silverstein, Ed. (Office of Naval Research ACR-92, 1962) pp. 933-945.

[5]G. Chertock, "Sound Radiation from Prolate Spheroids," J. Acoust. Soc. Am. 33, (1961) pp. 871-876.

I. THE HELMHOLTZ FORMULA

Consider the flow of a uniform stream about a slender body of revolution. In Fig. 1, the uniform - U_0 is in the negative

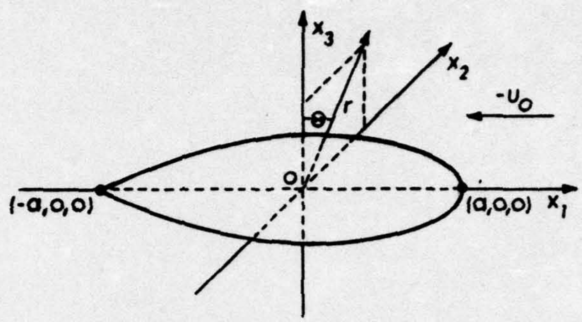

Fig. 1 - Coordinate System

x_1 -direction, and the body is placed with its longitudinal axis on the x-axis. For a compressible fluid, the velocity potential for the flow about such a body can be written as

$$\Phi_T = -U_0 x_1 + \Phi , \tag{1}$$

where

$U_0 x_1$ is the velocity potential of the uniform stream and

Φ is the velocity potential of the disturbance caused by the body. (The disturbance velocities are w_i, $i = 1, 2, 3$ with $w_i = +\partial\Phi/\partial x_i$.)

Then, for small vibrations of the body, the linearized equation for Φ is[6]

$$\left(1 - \frac{U_0^2}{c^2}\right)\frac{\partial^2 \Phi}{\partial x_1^2} + \frac{\partial^2 \Phi}{\partial x_2^2} + \frac{\partial^2 \Phi}{\partial x_3^2} + 2\,\frac{U_0}{c^2}\,\frac{\partial^2 \Phi}{\partial x_1 \partial t} = \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}, \tag{2}$$

where

c is the speed of sound and

t is time.

It will be assumed that the effect of the vibrating body can be approximated by changes in the flow caused by distributions of time-varying sources and doublets. The solutions of Eq. (2) for a source and a doublet are, respectively, the real parts of [7,8]

$$\Phi_s(x_1, x_2, x_3, t) = -\frac{m(\xi_1, \xi_2, \xi_3)}{4\pi R}\, e^{-ikc(t-D/c)} \tag{3}$$

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[6] I. E. Garrick, "Nonsteady Wing Characteristics," Section F of Aerodynamic Components of Aircraft at High Speeds, A. F. Anderson and H. R. Lawrence, Eds. (Princeton University Press, Princeton) p. 662.

[7] Reference 6, pp. 674-675.

[8] I. E. Garrick, "On Moving Sources in Nonsteady Aerodynamics and in Kirchhoff's Formula," Proceedings of the First U. S. National Congress of Applied Mechanics (The American Society of Mechanical Engineers, 1952) p. 735.

and

$$\Phi_D(x_1, x_2, x_3, t) = -\frac{\mu_n(\xi_1, \xi_2, \xi_3)}{4\pi} \frac{\partial}{\partial n} \left\{ \frac{e^{-ikc(t-D/c)}}{R} \right\}, \tag{4}$$

where

Φ_s, (Φ_D) is the velocity potential at point (x_1, x_2, x_3) and time t due to a source (doublet) at ξ_1, ξ_2, ξ_3;

$m(\xi_1, \xi_2, \xi_3)$ is the source strength (a source is a point of outward radial flow; its strength is the volume emitted per unit time);

$\mu_n(\xi_1, \xi_2, \xi_3)$ is the strength of the doublet with axis in a given direction n ;[9,10]

$$\frac{\partial}{\partial n} = \ell_1 \frac{\partial}{\partial \xi_1} + \ell_2 \frac{\partial}{\partial \xi_2} + \ell_3 \frac{\partial}{\partial \xi_3};$$

ℓ_1, ℓ_2, ℓ_3 are the direction cosines of the axis of the doublet;

$$R = \sqrt{(x_1 - \xi_1)^2 + \beta^2 [(x_2 - \xi_2)^2 + (x_3 - \xi_3)^2]} \; ;$$

$$D = \frac{M(x_1 - \xi_1) + R}{\beta^2};$$

$$M = \frac{U_0}{c};$$

$$\beta^2 = 1 - M^2;$$

λ is the wavelength; and

$$k = \frac{2\pi}{\lambda}.$$

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[9] H. Lamb, The Dynamical Theory of Sound (Edward Arnold Ltd., London, 1925, 2nd ed., Dover Publications, New York, 1960) p. 230.

[10] In general, m and μ are complex in Eqs. (3) and (4).

For harmonic time variation, let

$$\Phi(x_1, x_2, x_3, t) = \phi(x_1, x_2, x_3)\, e^{-ikct}. \tag{5}$$

Then, a representation for ϕ in the region external to the body, in terms of ϕ and its normal derivative on the body surface, is given by the following generalization of the Helmholtz formula:[11]

$$\phi(x_1, x_2, x_3) = \frac{1}{4\pi} \int_s \left\{ \left(-\frac{\partial \phi}{\partial n} \right) \frac{e^{ikD}}{R} + \phi \frac{\partial}{\partial n} \left(\frac{e^{ikD}}{R} \right) \right\} ds , \tag{6}$$

where

$\partial/\partial n$ is the derivative in the direction of the outward normal to S.

The force normal to ds is

$$F_n e^{-ikct} ds = (p - p_0)\, ds , \tag{7}$$

and the pressure p is given by

$$p - p_0 = + i \rho_0 kc\, \Phi + \rho_0 U_0 \frac{\partial \Phi}{\partial x_1}, \tag{8}$$

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[11] Reference 8, p. 737.

where

p_0 is the pressure in the undisturbed stream and

ρ_0 is the mass density of the fluid.

Thus,

$$\phi = -\frac{i}{\rho_0 kc} F_n + i\frac{M}{k}\frac{\partial \phi}{\partial x_1}, \tag{9}$$

and Eq. (6) can be written in the form

$$\phi = \frac{1}{4\pi} \int_s \left\{\left(-\frac{\partial \phi}{\partial n}\right)\frac{e^{ikD}}{R} + \frac{i}{k} M \frac{\partial \phi}{\partial \xi_1}\frac{\partial}{\partial n}\left(\frac{e^{ikD}}{R}\right) - \frac{i}{\rho_0 kc} F_n \frac{\partial}{\partial n}\left(\frac{e^{ikD}}{R}\right)\right\} ds. \tag{10}$$

But,

$$i\frac{M}{k}\frac{\partial \phi}{\partial \xi_1}\frac{\partial}{\partial n}\left(\frac{e^{ikD}}{R}\right) = \left[+\frac{M^2}{\beta^2}\ell_1 + \frac{M}{\beta^2}\frac{\partial R}{\partial n} + i\frac{M}{k}\frac{1}{R}\frac{\partial R}{\partial n}\right]\frac{\partial \phi}{\partial \xi_1}\frac{e^{ikD}}{R}. \tag{11}$$

Then, for $M << 1$ and the farfield where $R >> \lambda$, the second term of the integrand of Eq. (10) is small compared with the first term, and $\phi(x_1, x_2, x_3)$ can be approximated by

$$\phi = -\frac{1}{4\pi} \int_s \left\{ w_n \frac{e^{ikD}}{R} + \frac{i}{\rho_0 kc} F_n \frac{\partial}{\partial n}\left(\frac{e^{ikD}}{R}\right)\right\} ds, \tag{12}$$

where

$$w_n = +\frac{\partial \phi}{\partial n}.$$

For axially symmetric flow, the surface integrals in Eq. (12) can be approximated by single integrals along the axis of the body. The volume of fluid flow per unit time across an elementary ring area of the surface at the mean position of the body can be written as

$$m(\xi_1)\, e^{-ikct}\, d\xi_1 \,. \tag{13}$$

The resultant of the force F_n due to the ring area is directed along the x_1-axis and can be written as

$$F_1(\xi_1)\, e^{-ikct}\, d\xi_1 \,. \tag{14}$$

For points far from the body (i. e., terms proportional to $1/R^2$ are neglected), the following approximations hold:

$$\frac{e^{ikD}}{R} = \frac{e^{ik\beta^{-2}\left[M(x_1-\xi_1) + R_0 - \xi_1 \cos\theta_0\right]}}{R_0}\left[1 + i\,\epsilon_0\right], \tag{15}$$

$$\frac{\partial}{\partial \xi_1}\left(\frac{e^{ikD}}{R}\right) = -i\frac{k}{\beta^2}\left[\cos\theta_0 + M\right]\frac{e^{ik\beta^{-2}\left[M(x_1-\xi_1) + R_0 - \xi_1 \cos\theta_0\right]}}{R_0}\left[1 + i\,\epsilon_0\right], \tag{16}$$

where

$$R_0 = \sqrt{x_1^2 + \beta^2 r^2} \doteq \sqrt{x_1^2 + r^2}, \quad M << 1 ;$$

$$r^2 = x_2^2 + x_3^2 ;$$

$$\cos \theta_0 = x_1/R_0 ;$$

$$|\epsilon_0| \leq 2\pi b/\lambda ;$$

b is the radius of the body at $x_1 = 0$; and

ℓ is the length of the body ($\ell = 2a$) .

With the conditions that $M << 1$ and $K \ell M << 1$ the velocity potential Φ can be approximated by

$$\Phi = -\frac{1}{4\pi} \frac{e^{-ikc(t - D_0/c)}}{R_0} \int_{-a}^{a} \left[m(\xi_1) + \frac{1}{\rho_0 c} \cos \theta_0 \, F_1(\xi_1) \right] e^{-ik \cos \theta_0 \cdot \xi_1} d\,\xi_1, \tag{17}$$

where

$$D_0 = \frac{M x_1 + R_0}{\beta^2} .$$

With this expression for Φ, the pressure in the farfield is approximated (for $M << 1$) by the real part of

$$p - p_0 = + i \rho_0 kc \, \Phi . \tag{18}$$

A similar procedure can be applied for transverse vibrations. However, in Section III a different (but equivalent) method is used.

II. *LONGITUDINAL VIBRATION*

The farfield pressure is determined by Eqs. (17) and (18) in terms of the flow at the surface of the body. Let the equation for the undisturbed body of revolution be

$$r = b(a_1) . \tag{19}$$

For the axially symmetric longitudinal vibration, the horizontal displacement of a point on the body surface is given by the real part of

$$a_0(x_1, t) = i\, a_0(x_1)\, e^{-ikct}. \tag{20}$$

The special cases considered are:

$$\text{(a) rigid body motion, } a_0(x_1) = -a_0 \, , \; a_0 << b, \tag{21}$$

$$\text{(b) accordion motion, } a_0(x_1) = a_0 \sin\left(n \frac{\pi}{2} \frac{x_1}{a}\right). \tag{22}$$

The equation for the vibrating body is

$$f(x_1, \Theta, r, t) = r - b(x_1^*) = 0, \tag{23}$$

where

$x_1^* = x_1 - i\, a_0(x_1)\, e^{-ikct}$ and

x_1, Θ, r are the cylindrical coordinates shown in Fig. 1.

For the linearized problem with harmonic time variation, the axially symmetric velocity potential will be written as

$$\Phi(x_1, r, t) = \Phi_0(x_1, r) + \phi(x_1, r)\, e^{-ikct}. \tag{24}$$

The boundary condition at the body surface is

$$\frac{Df}{Dt} = w_r \frac{\partial f}{\partial r} + w_\Theta \frac{1}{r}\frac{\partial f}{\partial \Theta} + w_1 \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial t} = 0, \tag{25}$$

where

$w_r = + \partial\Phi/\partial r$,

$w_\Theta = + \partial\Phi/\partial\Theta = 0$, for axially symmetric flow, and

$w_1 = -U_0 + \partial\Phi/\partial x_1$.

From Eqs. (19) through (25), the boundary conditions for Φ_0 and ϕ at the mean position of the body are (neglecting second order terms)

$$\frac{\partial \Phi_0}{\partial r} = -U_0\, b'(x_1), \tag{26}$$

$$\frac{\partial \phi}{\partial r} = \left[+ i U_0\, a_0'(x_1) - k c\, a_0(x_1)\right] b'(x_1), \tag{27}$$

where

$$b'(x_1) = \frac{db(x_1)}{dx_1},$$

and

$$a_0'(x_1) = \frac{da_0(x_1)}{dx_1}.$$

The boundary conditions can be applied at the mean position of the body, provided the slope $b'(x_1)$ is of the order of $d/\ell \ll 1$ and the radius of curvature of the body profile is large compared with the maximum amplitude of vibration.

For rigid body motion, $a_0'(x_1) = 0$, and the linearized boundary condition given by Eq. (27) does not depend on U_0. For accordion motion, the relative importance of the two terms on the right-hand side of Eq. (27) depends essentially on the ratio[12]

$$\frac{U_0 n \frac{\pi}{2} \frac{a_0}{a}}{k c a_0} = \frac{1}{2} n M \frac{\lambda}{\ell}. \tag{28}$$

Thus, for $n = 1$, $m = 0.01$, $\lambda/\ell = 1$, the effect of the first term on the right-hand side of Eq. (27) will be negligible. Only for wavelengths much greater than the body length, combined possibly with a large n, would this term be significant when $M << 1$.

The velocity potential Φ_0 that satisfies the boundary condition (26) is determined approximately for a slender body by a distribution

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[12]See also Eq. (53).

of sources, along the axis of the body, with strength per unit length given by[13]

$$m_0(x_1) = -U_0 S'(x_1), \tag{29}$$

where

$S(x_1)$ is the sectional area of the body.

It will be assumed that the velocity potential $\phi(x_1, r)$ can be approximated at the body surface by

$$\phi(x_1, r) = -\frac{1}{4\pi}\int_{-a}^{a} m(\xi_1)\frac{e^{ikD_1}}{R_1}\,d\xi_1, \tag{30}$$

where

$$R_1 = \sqrt{(x_1 - \xi_1)^2 + \beta^2 r^2} \quad \text{and}$$

$$D_1 = \frac{M(x_1 - \xi_1) + R_1}{\beta^2}$$

This can be approximated by

$$\phi(x_1, r) = -[1 + 0(k_1^2\,\ell^2 M^2)]\frac{1}{4\pi}\int_{-a}^{a} m(\xi)[1 + ik_1 M(x_1 - \xi_1)]\frac{e^{ik_1 R_1}}{R_1}\,d\xi_1, \tag{31}$$

---

[13] S. Goldstein, Lectures on Fluid Mechanics (Interscience Publishers, Inc., New York, 1960), p. 183.

where

$k_1 = 1/\beta^2 k$, and

$y = 0(x)$ means $\lim_{x \to 0} \frac{y}{x}$ is bounded or zero.

The approximation requires that

$$k_1 \ell M = \frac{2\pi}{\beta^2} \left(\frac{\ell}{\lambda}\right)\left(\frac{U_0}{c}\right) << 1. \tag{32}$$

Thus, it is not necessary for the wavelength λ to be much greater than the body length ℓ, provided $M = U_0/c$ is sufficiently small. The radial velocity is

$$+\frac{\partial \phi(x_1, r)}{\partial r} = \frac{1}{4\pi} \beta^2 k_1^3 r \int_{-a}^{a} m(\xi_1)[1 + ik_1 M(x_1 - \xi_1)] \left\{ \frac{\cos k_1 R_1}{(k_1 R_1)^3} \right.$$

$$\left. + \frac{\sin k_1 R_1}{(k_1 R_1)^2} - i \left[\frac{\cos k_1 R_1}{(k_1 R_1)^2} - \frac{\sin k_1 R_1}{(k_1 R_1)^3} \right] \right\} d\xi_1 . \tag{33}$$

If the factor in braces is expanded in terms of $k_1 R_1$ then the asymptotic expansion for $\partial\phi/\partial r$ as r tends to zero is[14]

$$+\frac{\partial \phi(x_1, r)}{\partial r} = \left[1 + 0\left(\frac{r^2}{\lambda^2} \ell n \frac{r}{\lambda}\right)\right] \frac{1}{2\pi} \frac{m(x_1)}{r} . \tag{34}$$

[14] Reference 13, p. 186.

Then, from Eq. (27)

$$m_1(x_1) = -kca_0(x_1)S'(x_1), \tag{35}$$

and

$$m_2(x_1) = +U_0a_0'(x_1)S'(x_1), \tag{36}$$

where

$$\begin{aligned} m(x_1) &= m_1(x_1) + im_2(x_1) \quad \text{and} \\ S(x_1) &= \pi b^2(x_1). \end{aligned} \tag{37}$$

The asymptotic expansions used to determine $\partial\phi/\partial r$ for small r are not valid at the ends of the body unless the ends are cusped.[15] For the vibrating body, it will be assumed that the volume changes in small regions near the ends and the reaction on the fluid of these regions have a negligible effect on the farfield pressure compared with the effects due to the rest of the vibrating body.

Since the source distribution $m(x_1)$ specified by Eqs. (35) and (36) determines the flow across the mean surface of the vibrating body, it is also the source strength for Eq. (17) for the farfield pressure. With this source strength, the velocity potential $\phi(x_1, r)e^{-ikct}$, as determined by Eq. (30), reduces to just the first term on the right-hand side of Eq. (17) (under the same conditions

[15] Reference 13, p. 185.

used in determining Eq. (17)). In fact, within the approximations made, the source distribution completely determines the farfield. There is no net radial force since the flow is axially symmetric. Hence, the last two terms on the right-hand side of Eq. (17) are zero. The x_1 component of force on a ring element of area $2\pi\, b(x_1)\, dx_1$ *of the mean surface is per unit length*

$$F_1(x_1)\, e^{-ikct} = 2\pi b(x_1) \Big[p - p_0 \Big]_{r=b(x_1)} b'(x_1), \tag{38}$$

where

$$p - p_0 = \rho_0 \left[ikc\, \phi(x_1, r) + U_0 \frac{\partial \phi(x_1, r)}{\partial x_1} \right] e^{-ikct}.$$

It can be verified that $1/\rho_0 c\, F_1(x_1)$ can be neglected in Eq. (17) in comparison with $m(x_1)$.

III. TRANSVERSE VIBRATION

For the transverse vibration of a body of revolution in a uniform stream, the solution of Eq. (2) is the sum of the velocity potential due to the uniform stream and that due to the vibration. Let the transverse vibration be such that the circular sections of the body do not change shape or radius but vibrate with velocity $+W(x_1)\, e^{-ikct}$ perpendicular to the longitudinal axis. The flow near the body will be considered as represented by the superpositon of a transverse flow of velocity $-W(x_1) e^{-ikct}$ on the flow of the uniform stream past the undisturbed body. Since the perturbation must be small if the problem is to be linear, it will be assumed that the maximum amplitude δ of the vibration is such that $\delta << b$ and the change in shape of the meridian profile is continuous and of order $0(\delta/\ell)$. Then, with the additional conditions that d/ℓ, r/λ, U_0/c, $W/c << 1$, the solution of Eq. (2) that holds near the surface of the body is[16]

$$\phi(x_1, r, \Omega)\, e^{-ikct} = -\frac{1}{\pi} S(x_1)\, W(x_1) \frac{\cos\Omega}{r} e^{-ikct} [1 + 0(k^2 r^2 \ln kr)], \quad (39)$$

where

$W(x_1)$ is the transverse velocity of a section of the body and

$$\cos\Omega = \frac{X_3}{r}.$$

For incompressible flow this same solution holds with error factor $[1 + 0(d^2/\ell^2 \ln d/\ell)]$.[17] Hence, the flow can be considered as quasi-stationary (i.e., time-varying-incompressible) flow at the surface of the body. It can be represented by a distribution of three-dimensional doublets with axes perpendicular to the body axis and in the direction of $w(x_1)$. The strength of the doublet distribution per unit length along the axis is

$$\mu_3(x_1, t) = +2\, S(x_1)\, W(x_1)\, e^{-ikct}. \quad (40)$$

---

[16] J. W. Miles, "On Non-Steady Motion of Slender Bodies," Aeronaut. Quart. 2, November (1950) p. 186.

[17] B. Thwaites, Incompressible Aerodynamics (Oxford, Clarendon Press, 1960) p. 393.

For the flow of the uniform stream past the body there is a doublet distribution $\mu_1(x_1)$ along the axis with strength

$$\mu_1(x_1) = U_0 S(x_1) . \tag{41}$$

However, with the conditions imposed, the approximate solution of Eq. (2), as given by Eq. (38), does not depend on U_0, and the reaction of the body on the fluid is approximated by the reaction of the time-varying doublet distribution μ_3. Applying the extended Lagally theorem to this doublet distribution gives the reaction[18, 19]

$$F_3(x_1, t)\, dx_1 = -\rho_0 \frac{\partial}{\partial t} \mu_3(x_1, t)\, dx_1 . \tag{42}$$

$$= +2\, i\, \rho_0\, k c\, S(x_1)\, W(x_1)\, e^{-ikct}\, dx_1 . \tag{43}$$

The mutual interactions of the doublets would involve products of the velocities at different points on the body and are neglected in the linear theory.

For the slender body the force $F_3\ dx_1$ exerted on the fluid at each section acts as an isolated doublet with axis in the x_3-direction. The resulting farfield velocity potential is

$$\Phi(x_1, x_2, x_3, t) = -\frac{e^{-ikc(t - D_0/c)}}{4\pi R_0} \cos\theta_3 \left\{ 2\, ik \int_{-a}^{a} S(\xi_1)\, W(\xi_1)\, e^{-ik \cos\theta_0 \cdot \xi_1}\, d\xi_1 \right\} \tag{44}$$

where

$$\cos\theta_3 = \frac{x_3}{R_0}$$

[18] Reference 2, p. 18.

[19] L. Landweber and C. S. Yih, "Forces, Moments, and Added Masses for Rankine Bodies," J. Fluid Mech. 1, (1956) p. 332.

The farfield pressure is given by

$$p - p_0 = +\frac{1}{2\pi}\rho_0 k^2 c \cos\theta_3 \frac{e^{-ikc(t-D_0/c)}}{R_0}\int_{-a}^{a} W(\xi_1) S(\xi_1) e^{-ik\cos\theta_0\cdot\xi_1} d\xi_1 . \quad (45)$$

The same result is obtained if the velocity potential given by Eq. (39) is used in Eq. (12). In this case the two terms of Eq. (12) are equal. The first term gives the effect of the acceleration of the displaced fluid, and the second term gives the added mass effect in the approximating two-dimensional incompressible flow near the body.

IV. EXAMPLES OF AXIALLY SYMMETRIC VIBRATION

For axially symmetric vibration, the farfield pressure is given by

$$p - p_0 = -\frac{i}{4\pi}\rho_0 k c \frac{e^{-ikc(t-D_0/c)}}{R_0}\int_{-a}^{a} m(\xi_1) e^{-ik\cos\theta_0\cdot\xi_1} d\xi_1 , \quad (46)$$

where Eqs. (17) and (18) were used and the force term of Eq. (17) was neglected in accordance with the analysis indicated at the end of Section II. The types of vibration are specified by Eqs. (20) and (21). Using Eqs. (35) and (36) yields the corresponding source strengths:

(a) rigid body motion, $m_1(x_1) = +kca_0 S'(x_1)$, $m_2(x_1) = 0$, (47)

$$\text{(b) accordion motion, } m_1(x_1) = -k c a_0 \sin\left(\frac{n\pi}{2}\frac{x_1}{a}\right) S'(x_1), \tag{48}$$

$$m_2(x_1) = -\frac{n\pi}{2} U_0 \frac{a_0}{a} \cos\left(\frac{n\pi}{2}\frac{x_1}{a}\right) S'(x_1),$$

where

$m(x_1)$ is defined by Eq. (37),

and

$$S'(x_1) = \frac{dS(x_1)}{dx_1}.$$

It will be assumed that the slope of the sectional area curve can be represented by a finite sum of Legendre polynomials.[20] Thus,

$$S'(x_1) = \sum_{s=0}^{\bar{s}} A_s P_s\left(\frac{x_1}{a}\right), \tag{49}$$

where

the A_s are constants.

With the source strength defined by Eqs. (48) and (49), the integral in Eq. (46) is the sum of integrals of the form

$$\begin{aligned} P_s &= \int_{-a}^{a} \sin\left(\frac{n\pi}{2}\frac{\xi_1}{a}\right) P_s\left(\frac{\xi_1}{a}\right) e^{-ik\cos\theta_0 \cdot \xi_1}\, d\xi_1 \\ &= i(-i)^s\, a\,[j_s(u+v) - j_s(u-v)], \end{aligned} \tag{50}$$

---

[20] For the spheroid and streamline body examples considered in this section, $S'(x_1)$ is given exactly by (49) with only a few terms. For more general forms, $S'(x_1)$ can be approximated by (49).

and

$$Q_s = \int_{-a}^{a} \cos\left(\frac{n\pi}{2}\frac{\xi_1}{a}\right) P_s\left(\frac{\xi_1}{a}\right) e^{-ik\cos\theta_0 \cdot \xi_1}\, d\xi_1 \,. \tag{51}$$

$$= (-i)^s a\,[j_s(u+v) + j_s(u-v)]\,,$$

where

$u = ka\cos\theta_0$,

$v = n\pi/2$, and

$j_s(u)$ is the spherical Bessel function of order s.[21]

With these results, the farfield pressure for rigid body motion and accordion motion are, respectively, the real parts of

$$p - p_0 = -\frac{i}{2\pi}\rho_0 k^2 c^2 a\, a_0 \frac{e^{-ikc(t - D_0/c)}}{R_0} \sum_{s=0}^{\bar{s}} (-i)^s A_s j_s(u)\,, \tag{52}$$

and

$$p - p_0 = -\frac{1}{4\pi}\rho_0 k^2 c^2 a\, a_0 \frac{e^{-ikc(t - D_0/c)}}{R_0} \sum_{s=0}^{\bar{s}} (-i)^s A_s \Big\{[j_s(u+v) - j_s(u-v)] \tag{53}$$

$$+ \frac{1}{2} n M \frac{\lambda}{\ell}\,[j_s(u+v) + j_s(u-v)]\Big\}\,.$$

[21]P. M. Morse and H. Feshback, Methods of Theoretical Physics (McGraw-Hill Book Co., Inc., New York, 1953), p. 1573.

In the following examples it will be assumed that $nM(\lambda/\ell) << 1$, and the second terms of Eq. (53) will be neglected.

The equation of the meridian profile of a spheroid with center at the origin and axis of revolution along the x_1-axis is

$$r(x_1) = b\sqrt{1-\tilde{x}_1^2}, \tag{54}$$

where

$r^2 = x_2^2 + x_3^2$,

a is the semi-major axis,

b is the semi-minor axis, and

$\tilde{x}_1 = x_1/a$.

To illustrate the effect of a change in the distribution of volume along the axis of revolution, a simple family of streamline bodies can be constructed by adding the following function to the equation for the meridian profile of a spheroid:

$$r_1(x_1) = 2b\,\tilde{b}_1\,\tilde{x}_1\sqrt{1-\tilde{x}_1^2}, \tag{55}$$

where

$\tilde{b}_1 = b_1/b$ and

b_1 *is a constant that determines the amount of distortion from a spheroid.*

Figure 2 shows the meridian profile of a spheroid with $b/a = 0.1$ and the profiles of two streamline bodies with $b/a = 0.1$, $\tilde{b}_1 = 0.3$ and $b/a = 0.1$, $\tilde{b}_1 = 0.5$.

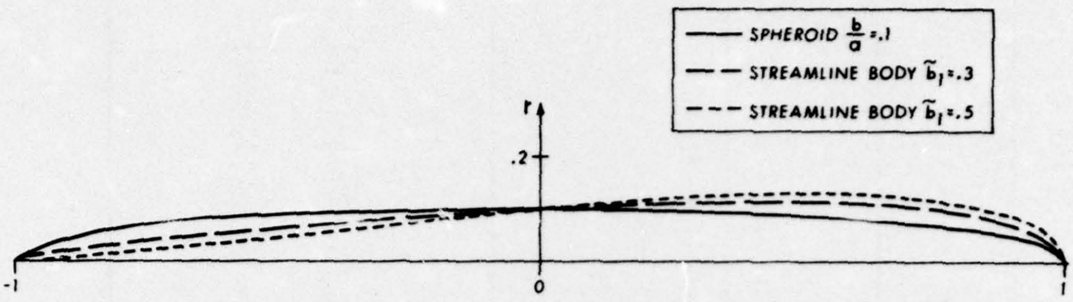

Fig. 2 - Meridian Profiles

The slope of the sectional area curves for the spheroid and the streamline bodies can be written as

$$S'(x_1) = -2\pi \frac{b^2}{a} P_1(\tilde{x}_1) \quad \text{(spheroid)}, \tag{56}$$

and

$$S'(x_1) = -\pi \frac{b^2}{a} \left[\left(2 + \frac{8}{5}\tilde{b}_1^2\right) P_1(\tilde{x}_1) + 8\tilde{b}_1 P_2(\tilde{x}_1) + \frac{32}{5}\tilde{b}_1^2 P_3(\tilde{x}_1) \right] \quad \text{(streamline body)}. \tag{57}$$

The real part of the farfield pressure determined by Eqs. (52) or (53) can be written as

$$(p - p_0)_{real} = \rho_0 k^2 c^2 b^2 a_0 \left\{ C_S \frac{\sin[kc(t - D_0/c)]}{R_0} + C_C \frac{\cos[kc(t - D_0/c)]}{R_0} \right\}, \tag{58}$$

where

C_s and C_c are non-dimensional functions of the parameters specifying the body and motion.

Table 1

THE FUNCTIONS C_s AND C_c

Type of Vibration \ Shape	Spheroid	Streamline Body
Rigid Body	$C_s = 0$	$C_s = -4\tilde{b}_1 j_2(u)$
	$C_c = j_1(u)$	$C_c = j_1(u) + \frac{4}{5}\tilde{b}_1^2 [j_1(u) - 4 j_3(u)]$
Accordion	$C_s = -\frac{1}{2}[j_1(u+v) - j_1(u-v)]$	$C_s = -\frac{1}{2}\left(1 + \frac{4}{5}\tilde{b}_1^2\right)[j_1(u+v) - j_1(u-v)]$ $+ \frac{8}{5}\tilde{b}_1^2 [j_3(u+v) - j_3(u-v)]$
	$C_c = 0$	$C_c = -2\tilde{b}_1 [j_2(u+v) - j_2(u-v)]$
Transverse		
$W_2 = 0,\ \Omega_1 = 1$	$C_s = 0$	$C_s = -4\tilde{b}_1 \cos\theta_3 J_2^+(u, 1)$
$a_0 = +a\frac{W_1}{c}$	$C_c = \cos\theta_3 J_1^+(u, 1)$	$C_c = \cos\theta_3 \left[\left(1 + \frac{4}{5}\tilde{b}_1^2\right) J_1^+(u, 1) - \frac{16}{5}\tilde{b}_1^2 J_3^+(u, 1)\right]$
$W_1 = 0,\ \Omega_2 = 1$	$C_s = \cos\theta_3 J_1^-(u, 1)$	$C_s = \cos\theta_3 \left[\left(1 + \frac{4}{5}\tilde{b}_1^2\right) J_1^-(u, 1) - \frac{16}{5}\tilde{b}_1^2 J_3^-(u, 1)\right]$
$a_0 = +a\frac{W_2}{c}$	$C_c = 0$	$C_c = 4\tilde{b}_1 \cos\theta_3 J_2^-(u, 1)$

For definition of J_s^+ and J_s^- see Eqs. (65) and (66)

Formulas for C_s and C_c for the spheroid and streamline body families with rigid body and accordion motion are given in Table 1. For the spheroid in rigid body vibration and the simplest case (i.e., $n = 1$) of accordion vibration these results agree with those of Strasberg,[3] who has shown that these results agree with those given by Chertock[5] for slender spheroids and $u < 3$. Since Chertock's results are based on an asymptotic expansion (for $b/\ell \to 0$) *of the exact solution for a spheroid, the condition* $u < 3$ *represents* a restriction on the results based on the slender body theory of the present report.

For a given body and mode of vibration, the independent variable in the formulas for C_s and C_c is $u = ka \cos \theta_0$. Thus, with ka held constant, the formulas show how the axially symmetric farfield pressure given by Eq. (58) depends on the position of the observation point as specified by $\cos \theta_0 = x_1/R_0$. For the spheroid in accordion motion both the body and the motion are symmetric with respect to x_1, and only a symmetric formula occurs. Similarly, for rigid body motion, which is not symmetric with respect to x_1, only a skewsymmetric formula occurs for the spheroid. The meridian profile of the streamline body is not symmetric with respect to x_1, and as a result one of the functions C_s, C_c is symmetric and one is skewsymmetric with respect to x_1 in each case. These functions are shown in Figs. 3 and 4. A further comparison of the radiated fields is provided if the average farfield intensity, based on the linear theory, is defined by

$$\Gamma = \frac{1}{\rho_0 c}\left[(p - p_0)_{real}\right]_{average} \qquad (59)$$

$$= \frac{1}{2}\rho_0 k^4 c^3 b^4 a_0^2 \cdot \frac{C_S^2 + C_C^2}{R_0^2},$$

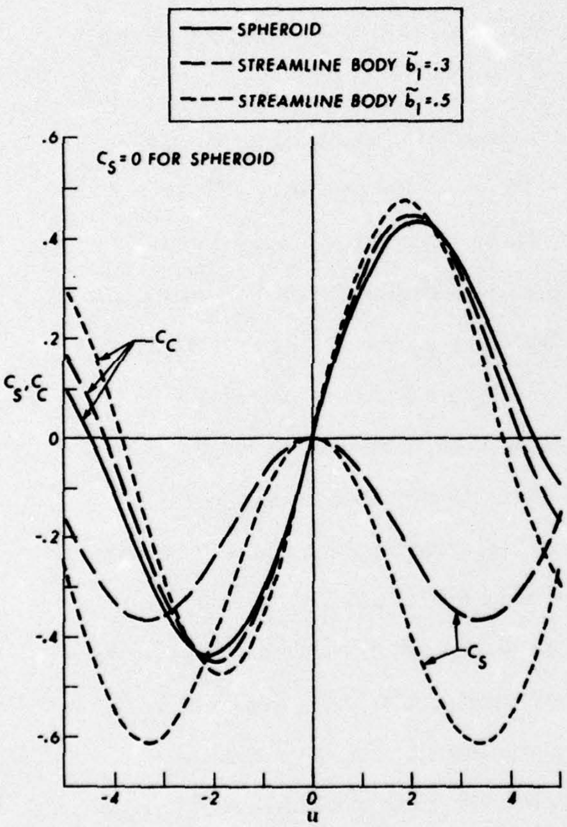

Fig. 3 - C_s and C_c Rigid Body Vibration

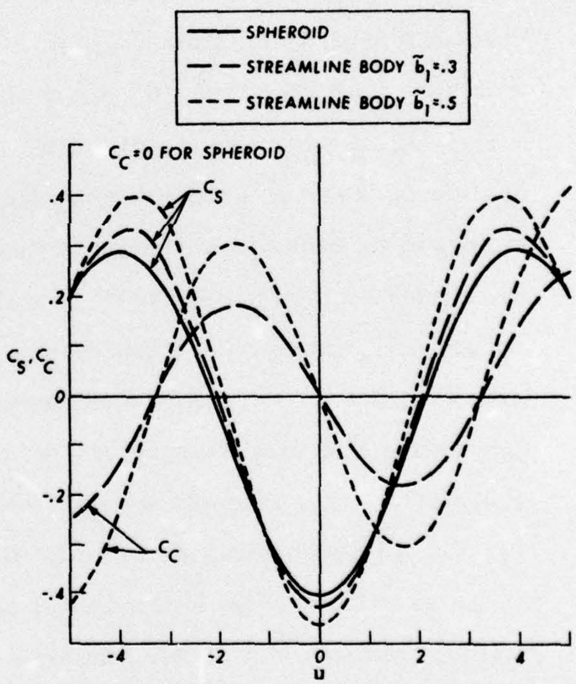

Fig. 4 - C_s and C_c Accordion Vibration

The function $C_s^2 + C_c^2$ is plotted in Figs. 5 and 6.

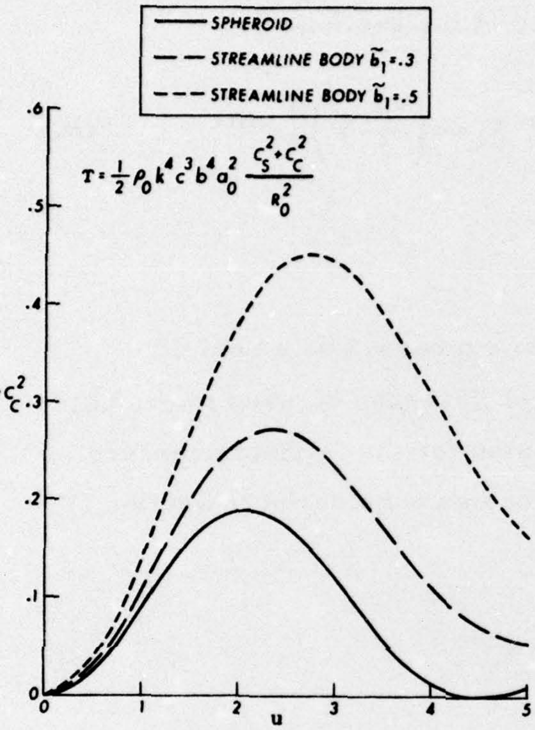

Fig. 5 - Intensity Rigid Body Vibration

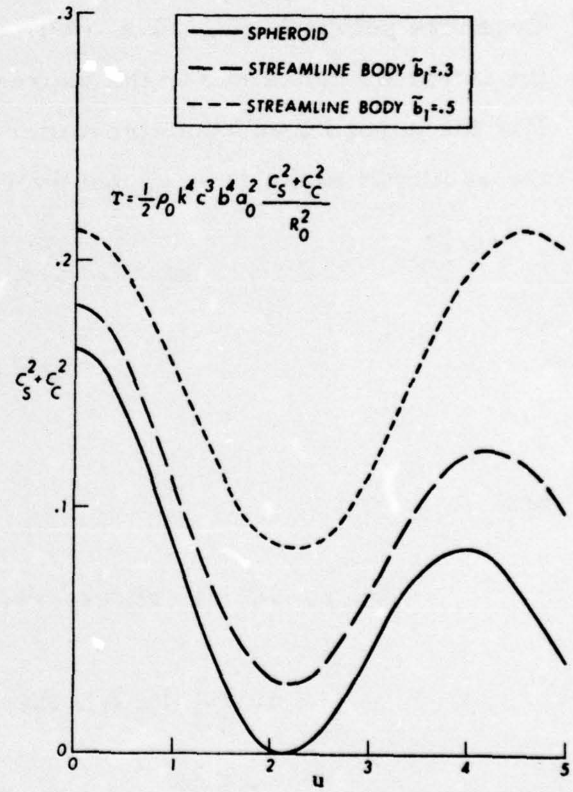

Fig. 6 - Intensity Accordion Vibration

V. EXAMPLES OF TRANSVERSE VIBRATION

For transverse vibration the farfield pressure is determined by Eq. (45). Let the transverse velocity of the sections be

$$W(x_1, t) = \left[W_1 \cos\left(\frac{\pi \Omega_1}{2} \tilde{x}_1\right) + W_2 \sin\left(\frac{\pi \Omega_2}{2} \tilde{x}_1\right)\right] e^{-ikct}, \tag{60}$$

where

W_1, W_2, Ω_1, and Ω_2 are constants.

If the sectional area of a given body is expressed as a sum of Legendre polynomials, Eqs. (50) and (51) can be used to evaluate the integrals appearing in the expression for the farfield pressure. For the spheroid and the streamline bodies considered in Section IV, the sectional areas are, respectively,

$$\begin{aligned} S(x_1) &= \pi b^2 (1 - \tilde{x}_1^2) \\ &= \pi b^2 \frac{2}{3} [P_0(\tilde{x}_1) - P_2(\tilde{x}_1)], \end{aligned} \tag{61}$$

and

$$\begin{aligned} S(x_1) &= \pi b^2 [(1 - \tilde{x}_1^2) + 4\tilde{b}_1 (\tilde{x}_1 - \tilde{x}_1^3) + 4\tilde{b}_1^2 (\tilde{x}_1^2 - \tilde{x}_1^4)] \\ &= \pi b^2 \left\{ \frac{2}{3} [P_0(\tilde{x}_1) - P_2(\tilde{x}_1)] - 4\tilde{b}_1 \frac{2}{5} [P_1(\tilde{x}_1) - P_3(\tilde{x}_1)] \right. \\ &\quad \left. + 4\tilde{b}_1^2 \left[\frac{24}{105} (P_2(\tilde{x}_1) - P_4(\tilde{x}_1)) + \frac{14}{105} (P_0(\tilde{x}_1) - P_2(\tilde{x}_1))\right]\right\}. \end{aligned} \tag{62}$$

The farfield pressures for the spheroid and the streamline bodies are, respectively,

$$p - p_0 = +\rho_0 k^2 c^2 b^2 \cos\theta_3 \frac{e^{-ikc(t - D_0/c)}}{R_0} \left[\left(a\frac{W_1}{c}\right) J_1^+(u, \Omega_1) + i\left(a\frac{W_2}{c}\right) J_1^-(u, \Omega_2)\right] \quad (63)$$

and

$$p - p_0 = +\rho_0 k^2 c^2 b^2 \cos\theta_3 \frac{e^{-ikc(t - D_0/c)}}{R_0} \left\{\left(1 + \frac{4}{5}\tilde{b}_1^2\right)\left(a\frac{W_1}{c}\right) J_1^+(u, \Omega_1)\right.$$

$$- \frac{16}{5}\tilde{b}_1^2 \left(a\frac{W_1}{c}\right) J_3^+(u, \Omega_1) + 4\tilde{b}_1 \left(a\frac{W_2}{c}\right) J_2^-(u, \Omega_2) + i\left[\left(1 + \frac{4}{5}\tilde{b}_1^2\right)\right. \quad (64)$$

$$\left.\left.\left(a\frac{W_2}{c}\right) J_1^-(u, \Omega_2) - \frac{16}{5}\tilde{b}_1^2 \left(a\frac{W_2}{c}\right) J_3^-(u, \Omega_2) - 4\tilde{b}_1 \left(a\frac{W_1}{c}\right) J_2^+(u, \Omega_1)\right]\right\},$$

where

$$J_s^+(u, \Omega_i) = \frac{j_s\left(u + \frac{\pi\Omega_i}{2}\right)}{\left(u + \frac{\pi\Omega_i}{2}\right)} + \frac{j_s\left(u - \frac{\pi\Omega_i}{2}\right)}{\left(u - \frac{\pi\Omega_i}{2}\right)}, \quad i = 1, 2; \quad (65)$$

and

$$J_s^-(u, \Omega_i) = \frac{j_s\left(u + \frac{\pi\Omega_i}{2}\right)}{\left(u + \frac{\pi\Omega_i}{2}\right)} - \frac{j_s\left(u - \frac{\pi\Omega_i}{2}\right)}{\left(u - \frac{\pi\Omega_i}{2}\right)}, \quad i = 1, 2. \quad (66)$$

If $W_2 = 0$ and $\Omega_1 = 1$, the maximum amplitude of vibration is at the half-length of the body, and there are nodes at the ends. Using the notation of Eq. (58) with $a_0 = - a\, W_1/c$ yields the expressions for C_s and C_c given in Table 1. For $W_1 = 0$, $\Omega_2 = 1$, the maximum amplitude of vibration occurs at the ends of the body, and there is a node at the half-length. Setting $a_0 = - a\, W_2/c$, the expressions for C_s and C_c are given in the last line of Table 1. Figures 7 and 8 show the functions $\tilde{C}_s$ and $\tilde{C}_c$ (where $C_s = \tilde{C}_s \cos\theta_3$ and $C_c = \tilde{C}_c \cos\theta_3$) for these examples of transverse vibration for spheroid and streamline body families. The corresponding intensities are given in Figs. 9 and 10.

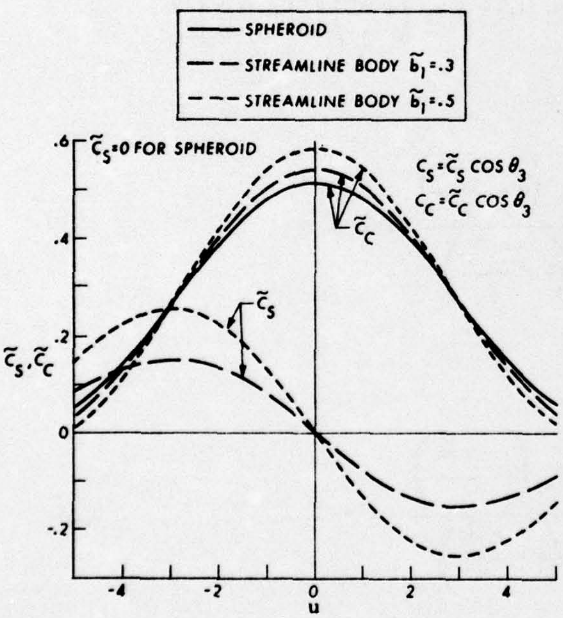

Fig. 7 - C_s and C_c Transverse Vibration
$W_2 = 0$, $\Omega_1 = 1$

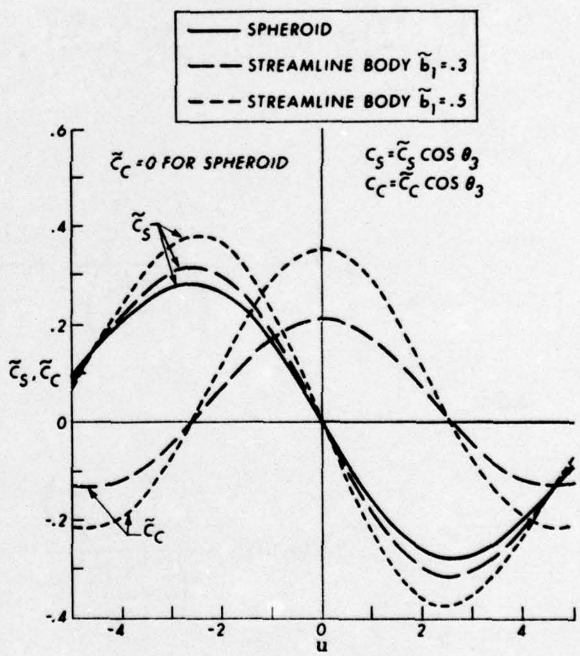

Fig. 8 - C_s and C_c Transverse Vibration
$W_1 = 0$, $\Omega_2 = 1$

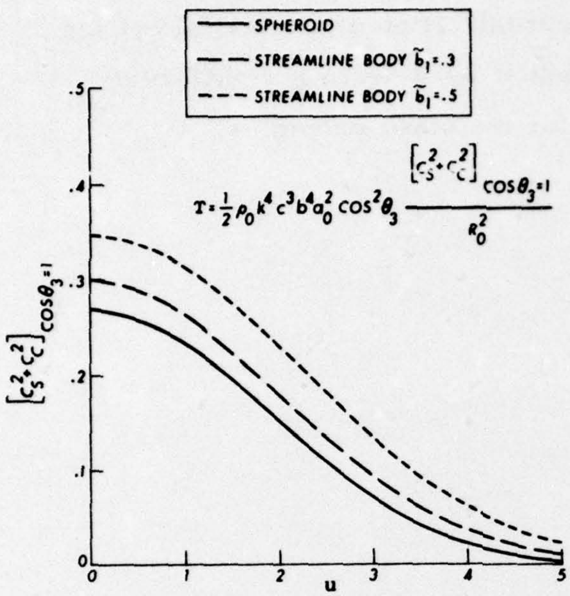

Fig. 9 - Intensity Transverse Vibration
$W_2 = 0, \Omega_1 = 1$

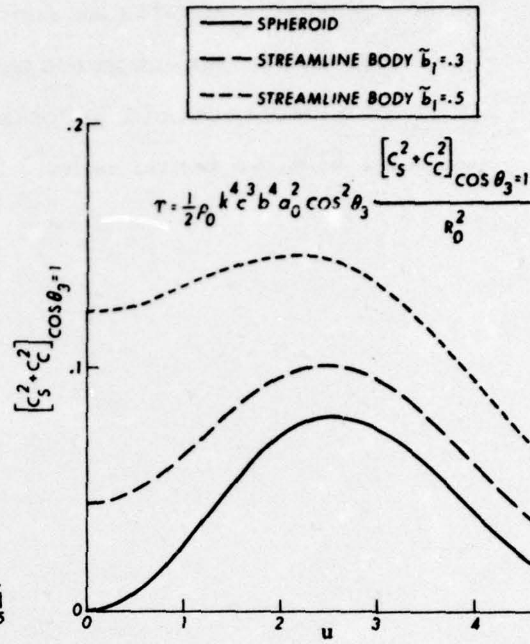

Fig. 10 - Intensity Transverse Vibration
$W_1 = 0, \Omega_2 = 1$

As a final remark on the comparison shown in Figs. 3-10, it should be noted that the streamline body family, as defined by Eqs. (54) and (55), was selected to give relatively simple expressions for the farfield pressure. All comparisons are for bodies with the same length $\ell = 2a$ and the same radius b at the center. This is illustrated by Fig. 2. It might be expected that the comparison should be for the same ratio of maximum diameter to length, or for the same volume, or some other specification of the spheroid to use in comparison with a streamline body. However, consider Fig. 3, which shows C_s and C_c for rigid body vibration. The spheroid provides a good approximation to C_c for the steamline bodies, but no spheroid provides an approximation for C_s. Thus, no significant improvement in the approximation of the farfield of the streamline bodies would be obtained by selecting a different spheroid. Similar remarks hold for the other examples.

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13. ABSTRACT

The radiated pressure field due to the low frequency vibration of a slender body of revolution is expressed in terms of a distribution of sources and doublets along the body axis. The strength of the singularities is determined from an analysis of the flow near the slender body. For axially symmetric flow a longitudinal rigid body vibration and a simple type of accordion vibration are considered. For these examples the source distribution has the dominant effect on the farfield pressure. For transverse vibration there is only a doublet distribution. The strength of the doublet distribution depends on the force the body exerts on the fluid. For wavelengths much greater than the maximum body diameter, this force can be conveniently determined by the extended Lagally theorem of incompressible hydrodynamics. Formulas for the farfield pressure for each type of vibration are given for spheroids and a simple class of streamline bodies. Examples illustrating the effect of change in body shape and type of vibration are given.

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Navy Underwater Sound Laboratory
Report No. 706
LOW FREQUENCY SOUND RADIATION FROM SLENDER BODIES OF REVOLUTION, by Hartley L. Pond.
7 February 1966. i-vi + 32 p., figs. UNCLASSIFIED

The radiated pressure field due to the low frequency vibration of a slender body of revolution is expressed in terms of a distribution of sources and doublets along the body axis. The strength of the singularities is determined from an analysis of the flow near the slender body. For axially symmetric flow a longitudinal rigid body vibration and a simple type of accordion vibration are considered. For these examples the source distribution has the dominant effect on the farfield pressure. For transverse vibration there is only a doublet distribution. The strength of the doublet distribution depends on the force the body exerts on the fluid. For wavelengths much greater than the maximum body diameter, this force can be conveniently determined by the extended Lagally theorem of incompressible hydrodynamics. Formulas for the farfield pressure for each type of vibration are given for spheroids and a simple class of streamline bodies. Examples illustrating the effect of change in body shape and type of vibration are given.

1. Bodies of revolution
2. Pressure fields
3. Sound – Radiation

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